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Neutrino-production of a charmed meson and the transverse spin structure of the nucleon

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We calculate the amplitude for exclusive neutrino production of a charmed meson on an unpolarized target, in the collinear QCD approach where generalized parton distributions (GPDs) factorize from perturbatively calculable coefficient functions. We demonstrate that the transversity chiral odd GPDs contribute to the transverse cross section if the hard amplitude is calculated up to order m_c/Q . We show how to access these GPDs through the azimuthal dependence of the $\nu N \rightarrow \mu^- D^+ N$ differential cross section.

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Introduction. The transverse spin structure of the nucleon - that is the way quarks and antiquarks spins share the polarization of a nucleon, when it is polarized transversally to its direction of motion - is almost completely unknown. The transversity distributions which encode this information have proven to be among the most difficult hadronic quantities to access. This is due to the chiral odd character of the quark operators which enter their definition; this feature enforces the decoupling of these distributions from most measurable hard amplitudes. After the pioneering works [1], much effort [2] has been devoted to the exploration of many channels but experimental difficulties have challenged the most promising ones.

It is now well established that generalized parton distributions (GPDs) give access to the internal structure of hadrons in a much more detailed way than parton distributions (PDFs) measured in inclusive processes. The study of exclusive reactions mediated by a highly virtual photon in the generalized Bjorken regime benefits of the factorization properties of the leading twist QCD amplitudes [3] for reactions such as deeply virtual Compton scattering. A welcome feature of this formalism is that spin related quantities such as helicity or transversity GPDs may be accessed in reactions on an *unpolarized* nucleon.

Neutrino production is another way to access (generalized) parton distributions [4]. Neutrino induced cross sections are orders of magnitudes smaller than those for electroproduction and neutrino beams are much more difficult to handle than charged lepton beams; nevertheless, they have been very important to scrutinize the flavor content of PDFs and the advent of new generations of neutrino experiments will open new possibilities. We want here to stress that they can help to access the elusive chiral-odd generalized parton distributions. Some effort has already been dedicated to this question within the domain of virtual photon mediated processes [5], but the main result is that the studies are likely to be out of the abilities of present accelerators; future electron-ion

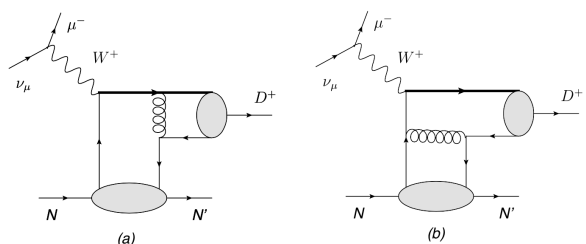


FIG. 1: Feynman diagrams for the factorized amplitude for the $\nu_\mu N \rightarrow \mu^- D^+ N'$ process; the thick line represents the heavy quark.

colliders [6] may help. The only exception is the proposal [7] that pion electroproduction data are sensitive to transversely GPDs thanks to their interplay with the chiral-odd twist 3 Distribution Amplitude (DA) of the pseudo scalar mesons.

In this paper we consider the exclusive reactions

$$\begin{aligned} \nu_l(k) N(p_1) &\rightarrow l^-(k') D^+(p_D) N'(p_2), \\ \bar{\nu}_l(k) N(p_1) &\rightarrow l^+(k') D^-(p_D) N'(p_2), \end{aligned} \quad (1)$$

in the kinematical domain where collinear factorization leads to a description of the scattering amplitude in terms of nucleon GPDs and the D -meson distribution amplitude, with the hard subprocess ($q = k' - k$; $Q^2 = -q^2$):

$$W^+(q) d \rightarrow D^+ d' \quad W^-(q) u \rightarrow D^- u', \quad (2)$$

described by the handbag Feynman diagrams of Fig. 1.

We will demonstrate that the transverse amplitude $W_T q \rightarrow D q'$ gets its leading term in the collinear QCD framework as a convolution of chiral odd leading twist GPDs with a coefficient function of order $\frac{m_c}{Q^2}$ (to be compared to the $O(\frac{1}{Q})$ longitudinal amplitude) and that it should be measurable in near future experiments at neutrino factories.

The azimuthal dependence of neutrino production. The dependence of a lepton production cross section on azimuthal angles is a well documented [8] and widely used way to analyze the scattering mechanism. This procedure is helpful as soon as one can define an angle φ between a leptonic and a hadronic plane, as for deeply virtual Compton scattering [9] and related processes. In the neutrino case, it reads :

$$\frac{d^4\sigma(\nu N \rightarrow l^- N' D)}{dx_B dQ^2 dt d\varphi} = \tilde{\Gamma} \left\{ \frac{1 + \sqrt{1 - \varepsilon^2}}{2} \sigma_{--} + \varepsilon \sigma_{00} \right. \\ \left. + \sqrt{\varepsilon(\sqrt{1 + \varepsilon} + \sqrt{1 - \varepsilon})} (\cos \varphi \operatorname{Re} \sigma_{-0} + \sin \varphi \operatorname{Im} \sigma_{-0}) \right\}, \quad (3)$$

with

$$\tilde{\Gamma} = \frac{G_F^2}{(2\pi)^4} \frac{1}{16x_B} \frac{1}{\sqrt{1 + 4x_B^2 m_N^2/Q^2}} \frac{1}{(s - m_N^2)^2} \frac{Q^2}{1 - \epsilon},$$

and the “cross-sections” $\sigma_{lm} = \epsilon_l^{*\mu} W_{\mu\nu} \epsilon_m^\nu$ are product of amplitudes for the process $W(\epsilon_l) N \rightarrow D N'$, averaged (sommé) over the initial (final) hadron polarizations. In the anti-neutrino case, one gets a similar expression with $\sigma_{--} \rightarrow \sigma_{++}$, $\sigma_{-0} \rightarrow \sigma_{+0}$, $1 + \sqrt{1 - \varepsilon^2} \rightarrow 1 - \sqrt{1 - \varepsilon^2}$ and $\sqrt{1 + \varepsilon} + \sqrt{1 - \varepsilon} \rightarrow \sqrt{1 + \varepsilon} - \sqrt{1 - \varepsilon}$. We use the standard notations of deep exclusive lepton production, namely $P = (p_1 + p_2)/2$, $\Delta = p_2 - p_1$, $t = \Delta^2$, $x_B = Q^2/2p_1 \cdot q$, $y = p_1 \cdot q / p_1 \cdot k$ and $\epsilon \simeq 2(1 - y)/[1 + (1 - y)^2]$. p and n are light-cone vectors ($v \cdot n = v^+$, $v \cdot p = p^-$ for any vector v) and $\xi = -\Delta \cdot n / 2P \cdot n$ is the skewness variable. The azimuthal angle φ is defined [8] in the initial nucleon rest frame as:

$$\sin \varphi = \frac{\vec{q} \cdot [(\vec{q} \times \vec{p}_D) \times (\vec{q} \times \vec{k})]}{|\vec{q}| |\vec{q} \times \vec{p}_D| |\vec{q} \times \vec{k}|}, \quad (4)$$

while the final nucleon momentum lies in the xz plane ($\Delta^y = 0$) and $\epsilon^{0123} = -1$.

We now focus on the evaluation of the longitudinal and transverse amplitudes which will (in the neutrino case) contribute respectively to σ_{00} and σ_{--} , while their interference will construct σ_{-0} . Two ingredients need first to be defined, namely the D -meson distribution amplitude and the transversity GPDs.

D-meson distribution amplitude. In the collinear factorization framework, the hadronization of the quark-antiquark pair is described by a distribution amplitude (DA) which obeys a twist expansion and evolution equations. Much work has been devoted to this subject [10]. Here, we shall restrict ourselves to a leading twist description of the D -meson DA, defined as (we omit the path-ordered gauge link):

$$\langle 0 | \bar{d}(y) \gamma^\mu \gamma^5 c(-y) | D(p_D) \rangle = i f_D P^\mu \int_0^1 e^{i(2z-1)p_D \cdot y} \phi_D(z),$$

where $\int_0^1 dz \phi_D(z) = 1$ and $f_D = 0.223 \text{ GeV}$.

Transversity GPDs. The twist 2 transversity GPDs have been defined [11] and their experimental access much discussed [5]. They correspond to the tensorial Dirac structure $\psi^q \sigma^{\mu\nu} \psi^q$. In the nucleon case, there are four twist 2 transversity GPD defined as:

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \langle p_2, \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | p_1, \lambda \rangle \Big|_{z^+ = z_T = 0} \\ = \frac{1}{2P^+} \bar{u}(p_2, \lambda') \left[H_T^q i\sigma^{+i} + \tilde{H}_T^q \frac{P^+ \Delta^i - \Delta^+ P^i}{m_N^2} \right. \\ \left. + E_T^q \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m_N} + \tilde{E}_T^q \frac{\gamma^+ P^i - P^+ \gamma^i}{m_N} \right] u(p_1, \lambda). \quad (5)$$

The leading GPD $H_T(x, \xi, t)$ is equal to the transversely PDF in the $\xi = t = 0$ limit, which has recently been argued [12] to be sizable and negative for the d -quark, which is contributing to the process under study here.

The longitudinal amplitude. The longitudinal leading twist leading order amplitude has been computed previously [13] for a pseudo scalar light meson and the calculation for the D -meson case is but a slight modification of this result. It is a convolution of chiral-even GPDs $H^d(x, \xi, t)$, $\tilde{H}^d(x, \xi, t)$, $E^d(x, \xi, t)$ and $\tilde{E}^d(x, \xi, t)$ and reads:

$$T_L = \frac{-iC}{Q} \bar{N}(p_2) \left[\mathcal{H}_D \hat{n} + \frac{1}{2m_N} \mathcal{E}_D i\sigma^{n\Delta} \right. \\ \left. - \tilde{\mathcal{H}}_D \hat{n} \gamma_5 - \frac{\Delta \cdot n}{2m_N} \tilde{\mathcal{E}}_D \gamma_5 \right] N(p_1), \quad (6)$$

with $C = \frac{8\pi}{9} \alpha_s V_{dc}$ and $(\bar{z} = 1 - z)$:

$$\mathcal{F}_D(\xi, t) = f_D \int dz \frac{\phi_D(z)}{\bar{z}} \int dx \frac{F^d(x, \xi, t)}{x - \xi + i\epsilon}, \quad (7)$$

for any chiral even d -quark GPD in the nucleon $F^d(x, \xi, t)$; g is the weak interaction coupling constant and V_{dc} the CKM matrix element.

The transverse amplitude up to $O(m_c/Q^2)$. It is straightforward to show that the transverse amplitude vanishes at the leading twist level. For chiral-even GPDs, this comes from the collinear kinematics appropriate to the calculation of the leading twist coefficient function; for chiral-odd GPDs, this comes from the odd number of γ matrices in the Dirac trace.

To estimate the transverse amplitude, one thus needs to evaluate next to leading twist contributions. This is quite a hard task if one wants to get all contributions, and we shall restrict ourselves to a self-consistent (and gauge invariant) part, namely the heavy quark mass corrections in the coefficient function. Indeed, it has been demonstrated [14] that hard-scattering factorization is valid with the inclusion of heavy quark masses in the hard amplitude. This proof is applicable independently of the relative sizes of the heavy quark masses and Q , and the size of the errors is a power of Λ/Q independently

of the mass scale. In our case, this means including the part $\frac{m_c}{k_c^2 - m_c^2}$ in the off-shell heavy quark propagator in the Feynman graph depicted on Fig. 1a. We keep the term in m_c^2 in the denominator since it will help us to understand precisely how to perform the integration around the point $x = \xi$. Adding this part of the heavy quark propagator has no effect on the calculation of the longitudinal amplitude (because of the odd number of γ matrices in the Dirac trace) but leads straightforwardly to a non-zero transverse amplitude when a chiral-odd transversity GPD is involved.

In the Feynman gauge, the non-vanishing m_c -dependent part of the Dirac trace in the hard scattering part depicted in Fig. 1a reads:

$$\begin{aligned} \text{Tr} [\sigma^{pi} \gamma^\nu \hat{p}_D \gamma^5 \gamma_\nu \frac{m_c}{k_c^2 - m_c^2} (1 - \gamma^5) \hat{\epsilon} \frac{1}{k_g^2}] \\ = \frac{2Q^2}{\xi} \epsilon_\mu [i\epsilon^{\mu pin} - g_\perp^{\mu i}] \frac{m_c}{k_c^2 - m_c^2 + i\epsilon} \frac{1}{k_g^2 + i\epsilon}, \end{aligned} \quad (8)$$

where k_c (k_g) is the heavy quark (gluon) momentum and ϵ the polarization vector of the W -boson (we denote $\hat{p} = p_\mu \gamma^\mu$ for any vector p). The fermionic trace vanishes for the diagram shown on Fig. 1b thanks to the identity $\gamma^\rho \sigma^{\alpha\beta} \gamma_\rho = 0$. The denominators of the propagators read:

$$\begin{aligned} k_c^2 - m_c^2 &= \frac{Q^2 + m_c^2}{2\xi} (x - \xi), \\ k_g^2 &= \bar{z} [\bar{z} m_c^2 + \frac{Q^2 + m_c^2}{2\xi} (x - \xi)]. \end{aligned} \quad (9)$$

The transverse amplitude is then written as ($\tau = 1 - i2$):

$$\begin{aligned} T_T &= \frac{iC\xi m_c}{\sqrt{2}Q^2} \bar{N}(p_2) \left[\mathcal{H}_T^\phi i\sigma^{n\tau} + \tilde{\mathcal{H}}_T^\phi \frac{\Delta^\tau}{m_N^2} \right. \\ &\quad \left. + \mathcal{E}_T^\phi \frac{\hat{n}\Delta^\tau + 2\xi\gamma^\tau}{2m_N} - \tilde{\mathcal{E}}_T^\phi \frac{\gamma^\tau}{m_N} \right] N(p_1), \end{aligned} \quad (10)$$

in terms of transverse form factors that we define as :

$$\mathcal{H}_T^\phi = f_D \int \frac{\phi(z)dz}{\bar{z}} \int \frac{H_T^d(x, \xi, t)dx}{(x - \xi + i\epsilon)(x - \xi + \alpha\bar{z} + i\epsilon)} \quad (11)$$

where H_T^d is any d-quark transversity GPD, $\alpha = \frac{2\xi m_c^2}{Q^2 + m_c^2}$ and we shall denote $\mathcal{F}_T = \xi\mathcal{E}_T - \tilde{\mathcal{E}}_T$.

A remark may be done about the calculation of the transverse form factors of Eq.11. Had we neglected the $\alpha\bar{z}$ term, we would have a double pole structure which is undefined if the derivative of the GPD is not continuous at the $x = \xi$ point, which indeed occurs in some models which do not contradict any basic principle. Simple algebra leads for the imaginary part of Eq.11:

$$\text{Im}\mathcal{H}_T = -\pi f_D \int dz \frac{\phi(z)}{\bar{z}} \frac{H_T^d(\xi, \xi, t) - H_T^d(\xi - \alpha\bar{z}, \xi, t)}{\alpha\bar{z}},$$

which has a legitimate limit provided the GPD has a continuous derivative on the left of the point ($x = \xi, \xi$).

Observables. We now calculate from T_L and T_T the quantities σ_{00} , σ_{--} and σ_{-0} which enter into the observables defined by Eq.3. The longitudinal cross section σ_{00} is straightforwardly obtained by squaring the amplitude T_L ; at zeroth order in Δ_T , it reads :

$$\sigma_{00} = \frac{C^2}{2Q^2} \left\{ 8(|\mathcal{H}_D^2| + |\tilde{\mathcal{H}}_D^2|)(1 - \xi^2) + |\tilde{\mathcal{E}}_D^2| \frac{1 + \xi^2}{1 - \xi^2} \right\} \quad (12)$$

At zeroth order in Δ_T , σ_{--} reads:

$$\begin{aligned} \sigma_{--} &= \frac{4\xi^2 C^2 m_c^2}{Q^4} \left\{ (1 - \xi^2) |\mathcal{H}_T^\phi|^2 + \frac{\xi^2}{1 - \xi^2} |\mathcal{F}_T^\phi|^2 \right. \\ &\quad \left. - 2\xi \mathcal{R}e[\mathcal{H}_T^\phi \mathcal{F}_T^{\phi*}] \right\}. \end{aligned} \quad (13)$$

The interference cross section σ_{-0} vanishes at zeroth order in Δ_T , so we give its expression at first order in Δ_T/m_N ; it reads (with $\lambda = \tau^* = 1 + i2$):

$$\begin{aligned} \sigma_{-0} &= \frac{-\xi\sqrt{2}C^2 m_c}{m_N} \frac{1}{Q^3} \left\{ -i\mathcal{H}_T^{*\phi} \tilde{\mathcal{E}}_D \xi(1 + \xi) \epsilon^{pn\Delta\lambda} \right. \\ &\quad + \mathcal{H}_T^{*\phi} \Delta^\lambda [-(1 + \xi)\mathcal{E}_D] + \tilde{\mathcal{H}}_T^{*\phi} \Delta^\lambda [2\mathcal{H}_D - \frac{2\xi^2}{1 - \xi^2} \mathcal{E}_D] \\ &\quad + \mathcal{E}_T^{*\phi} \Delta^\lambda [(1 - \xi^2)\mathcal{H}_D - \xi^2 \mathcal{E}_D] \\ &\quad \left. + \mathcal{F}_T^{*\phi} [\Delta^\lambda [(1 + \xi)\mathcal{H}_D + \xi\mathcal{E}_D] + i(1 + \xi) \epsilon^{pn\Delta\lambda} \tilde{\mathcal{H}}_D] \right\}. \end{aligned} \quad (14)$$

Let us finally estimate the magnitudes of the quantities $\mathcal{R}e(\sigma_{-0})$ and $\mathcal{I}m(\sigma_{-0})$ which are directly related to the observables $\langle \cos\varphi \rangle$ and $\langle \sin\varphi \rangle$ through

$$\begin{aligned} \langle \cos\varphi \rangle &= \frac{\int \cos\varphi \, d\varphi \, d^4\sigma}{\int d\varphi \, d^4\sigma} = K_\epsilon \frac{\mathcal{R}e\sigma_{-0}}{\sigma_{00}}, \\ \langle \sin\varphi \rangle &= K_\epsilon \frac{\mathcal{I}m\sigma_{-0}}{\sigma_{00}}, \end{aligned} \quad (15)$$

with $K_\epsilon = \frac{\sqrt{1+\epsilon} + \sqrt{1-\epsilon}}{2\sqrt{\epsilon}}$ and where we consistently neglected the $O(\frac{m_c^2}{Q^2})$ contribution of σ_{--} in the denominator. It should be noted that since Eq. 12 has a legitimate limit for small α , the dependence on the heavy meson DA effectively factorizes in the transverse form factors \mathcal{H}_T^ϕ , \mathcal{E}_T^ϕ , $\tilde{\mathcal{H}}_T^\phi$, $\tilde{\mathcal{E}}_T^\phi$ as it does in \mathcal{F}_D (Eq. 7), and thus disappears in the ratios of Eq. 15. If we take for granted that the $\mathcal{H}(\xi, t)$ form factor dominates among the chiral even form factors, we get quite simple approximate results:

$$\begin{aligned} \langle \cos\varphi \rangle &\approx \frac{K\mathcal{R}e[\mathcal{H}_D(2\tilde{\mathcal{H}}_T + (1 - \xi^2)\mathcal{E}_T + (1 + \xi)\mathcal{F}_T)^*]}{8|\mathcal{H}_D^2|}, \\ \langle \sin\varphi \rangle &\approx \frac{K\mathcal{I}m[\mathcal{H}_D(2\tilde{\mathcal{H}}_T + (1 - \xi^2)\mathcal{E}_T + (1 + \xi)\mathcal{F}_T)^*]}{8|\mathcal{H}_D^2|}, \\ K &= -\frac{\sqrt{1+\epsilon} + \sqrt{1-\epsilon}}{2\sqrt{\epsilon}} \frac{2\sqrt{2}\xi m_c}{Q} \frac{\Delta_T}{m_N}. \end{aligned} \quad (16)$$

In our kinematics, $\Delta^1 = \Delta^x = \Delta_T$, $\Delta^y = 0$, $\epsilon^{pn\Delta\lambda} = -i\Delta_T$.

Conclusion. We thus have defined a new way to get access to the transversity chiral-odd generalized parton distributions, the knowledge of which would shed a new light on the quark structure of the nucleons. Our main results are

- Collinear QCD factorization allows to calculate neutrino production of D -mesons in terms of GPDs.
- Chiral-odd and chiral-even GPDs contribute to the amplitude for different polarization states of the W (Eq.6 and Eq.11).
- The azimuthal dependence of the cross section allows to get access to chiral-odd GPDs (Eq.3).
- There is no small factor preventing the measurement from being feasible, provided ξ , $\frac{m_c}{Q}$, $\frac{\Delta_T}{m_N}$ are not too small (Eq.14 and Eq.16).

Planned high energy neutrino facilities [15] which have their scientific program oriented toward the understanding of neutrino oscillations or elusive inert neutrinos may thus allow - without much additional equipment - some important progress in the realm of hadronic physics. We do not claim that the experimental measurement of the observables proposed in Eq.15 and Eq.16 will be an easy task. One may advocate that the factor K in Eq. 16 is not small if we focus on Q in the range of 1 – 3 GeV and $\Delta_T/m_N \approx 0.5$. However, one may anticipate that the measurement of φ will be difficult since the reconstruction of the D -meson will not be complete, and that the exclusivity of the reaction will not be easy to prove since a neutrino beam has a wide energy spread and the target nucleon is inside a nucleus. A dedicated feasibility study is thus obviously needed to decide whether the observables defined here can be experimentally measured in a definite experimental set-up, but this is not within the goal of the present paper.

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